

Modeling Annuity Demand

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Agenda for today

- Basic Definitions.
- Generalize conditions for full-annuitization: Davidoff, Brown and Diamond (AER-2005).
- The Annuity Puzzle and the need to extend standard life-cycle model:
 - ① Bequest motives (Lockwood-2012-RED):
 - ② Various factors (Paschenko-2013-JPubE).

What is an annuity?

- An insurance product that pays out a fixed stream of payments to an individual as long as she is alive.
- Can be purchased with a lump sum or a series of payments and begin paying out almost immediately or at some point in the future.
- Is typically used as an income stream for retirees.

Generalizing Yaari (1965): Davidoff, Brown and Diamond (2005)

- Sufficient conditions for the optimality of full-annuitization in a very general setting
- Full annuitization might not be optimal if there are liquidity constraints or bequest motives.

DBD (2005) - Complete markets framework

- 2 periods model.
- Preferences: $U(c_1, c_2)$.
- Consumer is alive in period 2 with probability $1 - q$
- No bequest motive.
- Two securities available with different payoffs in period 2:
 - 1 A bond that returns R_B .
 - 2 An annuity that returns R_A if the consumer is alive.

DBD (2005) - Complete markets framework

- An actuarially fair annuity yields: $R_A = \frac{R_B}{1-q}$.
- While an annuity pays a consumer as long as she is alive and a bond pays no matter what (risk-free), it is natural to assume $R_A > R_B$.
- The problem to solve is:

$$\begin{aligned} \underset{c_1, A, B}{\text{Min}} \quad & c_1 + A + B \\ \text{s.t.} \quad & \end{aligned}$$

$$U(c_1, R_A A + R_B B) \geq \bar{U}$$

$$B \geq 0, \quad A \geq 0$$

- Annuities offer a higher return and there are not bequest motives: full annuitization.

Complete markets - Remark

- With complete markets, the result of full annuitization extends to:
 - ▶ Many periods.
 - ▶ Actuarially unfair annuities but still with $R_A > R_B$.
 - ▶ Intertemporally dependent utility that not need satisfy the expected utility axioms.

Incomplete markets

Two types of market incompleteness:

- Set of annuities restricted to the ones paying a constant real stream of income (Incomplete annuity markets). Two cases:
 - ① All trade occurs at once.
 - ② There are additional opportunities to trade.
- Uninsured expenses and illiquid annuities (Incomplete Securities market).

Incomplete annuity markets and all trade occurs at once

- Constant real annuity. Define by ι a row vector of ones with a length equal to the maximum length of life minus 1.
- Set of bonds is characterized by a vector of returns R_B .
- R_A is a vector of annuity payouts multiplying the scalar A to define state-by-state payouts.
- Assume that for any annuitized asset A and any collection of conventional assets B :

$$R_A A = R_B B \quad \rightarrow \quad A < \iota B$$

- For instance, if an annuity costs 1 unit of consumption in first period and pays R_{A2} and R_{A3} in following periods, then:

$$1 < R_{A2}/R_{B2} + R_{A3}/R_{B3}$$

Incomplete annuity markets and all trade occurs at once

- Consider a three-period setting, with bonds and a single available annuity and only a opportunity to trade after:

$$\underset{c_1, A, B}{\text{Min}} \quad c_1 + B_2 + B_3 + A$$

s.t.

$$U(c_1, R_{B2}B_2 + R_{A2}A, R_{B3}B_3 + R_{A3}A) \geq \bar{U}$$

$$B_2, B_3 \geq 0$$

- Given our assumption about returns, some annuitization is optimal and the optimum has zero bonds in at least one dated event.

Incomplete annuity markets and all trade occurs at once

- The consumer would want to buy bonds if at least one of the following conditions hold:

$$U_1(c_1, R_{A2}A, R_{A3}A) < R_{B2} U_2(c_1, R_{A2}A, R_{A3}A)$$

or

$$U_1(c_1, R_{A2}A, R_{A3}A) < R_{B3} U_3(c_1, R_{A2}A, R_{A3}A)$$

By the return assumption we cannot satisfy both of these conditions at the same time, but we might satisfy one of them.

Incomplete annuity markets with additional trade opportunities

- Additional trade opportunities into the 3-periods model by allowing the consumer to have savings at the end of the second period ($Z \geq 0$).
- Z has a return $R_Z = \frac{R_{B3}}{R_{B2}}$. Then:

$$\text{Min}_{c_1, A, B, Z} \quad c_1 + B_2 + B_3 + A$$

s.t.

$$U(c_1, R_{B2}B_2 + R_{A2}A - Z, R_{B3}B_3 + R_{A3}A + (R_{B3}/R_{B2})Z) \geq \bar{U}$$

$$R_{B2}B_2 + (R_{B2}/R_{B3})R_{B3}B_3 \geq 0$$

$$R_{B3}B_3 + (R_{B3}/R_{B2})Z \geq 0$$

Incomplete annuity markets with additional trade opportunities

Dissaving after full annuitization would not be attractive if:

$$R_{B2} U_2(c_1, R_{A2}A, R_{A3}A) \leq R_{B3} U_3(c_1, R_{A2}A, R_{A3}A)$$

If assumption about returns hold, the equation above is sufficient for full annuitization of initial savings:

$$R_{B3} < R_{A2}R_Z + R_{A3}$$

Also

$$R_{B2} U_2 < R_{A2} U_2 + (R_{A3}/R_{B3}) R_{B2} U_2 \leq R_{A2} U_2 + R_{A3} U_3$$

which is inconsistent with the FOC for positive holdings of both A and B_2 .

Incomplete securities and annuity markets: the role of liquidity

- There may be a non-insurable expense in the future.
- With incomplete markets, arbitrage-like dominance argument will not hold if bonds are liquid and annuities not.

Annuities and medical expenditures in the first period

- Uninsurable medical expense in the first period.
- This expenditure enters only into the budget constraint.
- Bonds can be sold in the first period with an early redemption penalty, but annuities cannot be sold:

$$\text{Min}_{c_1, A, B, I} c_1 + A + B + I$$

s.t.

$$(1 - m)U(c_1, R_A A + R_B B) + mU(c_1 - M + \beta I + \alpha_B B, R_A A) \geq \bar{U}$$

- Some bonds might be desirable if:
 - 1 Small difference between annuity and bonds return.
 - 2 Small early withdrawal penalty.
 - 3 Sufficiently actuarially unfair medical insurance pricing.

The annuity puzzle

Table 3

Summary statistics of the sample of 65–69-year-old single retirees used to estimate the demand for annuities.

Variable	Mean
Female	0.73
Age	67.1
Non-annuity wealth	\$271,209
Income	\$19,689
Any annuity	0.043
Life annuity	0.036
Have children	0.90
<i>N</i>	794

Notes. Statistics are raw (unweighted) means. I select retirees by dropping people with more than \$3000 in earnings in the year of the survey. The wealth variable includes all non-annuity wealth. The income variable refers to non-asset income. The “Any annuity” variable includes all private (non-pension) annuities. The “Life annuity” variable counts only those annuities that last for life. I assume that two-thirds of the individuals who own annuities of unknown type (because of missing variable values) own life annuities, which is the share of life annuities in total annuities for 65–69-year-olds whose annuity type can be determined.

Bequest motives - Lockwood (2011-RED)

- Numerical life cycle model to answer:
 - ① How strong must bequest motives be to eliminate purchases of available (actuarially unfair) annuities?
 - ② How many people would buy available annuities if everyone had one of several bequest motives estimated in the literature?
- Under plausible bequest motives and at available rates, people are likely going to be better off not annuitizing any wealth.
- **Punchline:** Bequest motives complement adverse selection!

Lockwood 2011 - Model

- Individual lives 2 periods with probability p .
- How much to consume, save and annuitize is decided in the first period:

$$c_1 + s + \pi = w \quad s \geq 0, \quad \pi \geq 0$$

- In old age, individual receives income y and his accumulated savings.
- Bequests if the individual dies young and wealth in old age are:

$$b_1 = Rs = R(w - c_1) - R\pi$$

$$x_2 = Rs + R_a\pi + y = R(w - c_1) + (R_a - R)\pi + y$$

Lockwood 2011 - Model

- In old age, wealth is split between consumption and an immediate bequest: $c_2 + b_2 = x_2$.
- Annuities allows the individual to trade short-lifespan bequests for wealth and vice-versa.
- Assume the following preferences:

$$EU = u(c_1) + \beta[pV(x_2) + (1 - p)v(b_1)]$$

where

$$V(x) = \text{Max}_{c \in [0, x]} u(c) + v(x - c)$$

$u(\cdot)$ and $v(\cdot)$ satisfy usual assumptions.

Lockwood 2011 - Model

- Marginal utility of annuitizing an additional unit of saving is given by:

$$\frac{\partial EU(c_1, \pi)}{\partial \pi} = \beta[\rho(R_a - R)V'(x_2) - (1 - \rho)Rv'(b_1)]$$

Write it as:

$$\frac{\partial EU(c_1, \pi)}{\partial \pi} = \beta R[(1 - \rho - \lambda)V'(x_2) - (1 - \rho)v'(b_1)]$$

where $R_a = (1 - \lambda)\frac{R}{p}$ and $\lambda \geq 0$ is the load.

- When $\lambda = 0$ (actuarially fair):

$$v'(b_1) = v'(b_2) \quad \text{then} \quad b_1^* = b_2^* = R(w - c_1^* - \pi^*)$$

$$c^* = R_a \pi^* + y$$

- With fair annuities, people set aside what they wish to bequeath and annuitize all future consumption ($R_a > R$).

Actuarially unfair annuities ($\lambda > 0$)

- With $\lambda > 0$ and bequest motives, people no longer fully annuitize planned future consumption:

$$V'(x_2) > v'(b_1)$$

which implies:

$$b_2^* < b_1^* \quad \text{and} \quad c_2^* > R_a \pi^* + y \quad \text{if} \quad b_1^* > 0$$

- Large enough loads can eliminate annuity purchases even by people who wish to consume more than their endowed income in old age:

$$\frac{\partial EU(c_1^*, \pi = 0)}{\partial \pi} = \beta R[(1-p-\lambda)V'(R(w-c_1^*)) - (1-p)v'(R(w-c_1^*))]$$

$$< 0$$

- How are loads on annuities in the data?

Bequest motives and the value of annuities

For people who wish to consume more than their pre-existing income, annuities:

- 1 Increase consumption at the expense of bequests.
- 2 Smooth consumption.
- 3 Insure bequests.

Simulations

Three exercises:

- 1 How bequest motives affect the value of annuities.
- 2 Decompose gains from annuities into its components.
- 3 Simulate the demand for annuities among single retirees in the U.S. using several estimates of bequest motives.

Simulations - Baseline model

- Life cycle model of retirement from age 65 until death.
- At age 65, a once-and-for-all choice about how much wealth to annuitize is made:

$$EU = \sum_{t=65}^T \beta^{t-65} S_t u(c_t) + \sum_{t=66}^{T+1} p_t v \left(\frac{b_t}{(1+r)^{t-65}} \right) \quad s.t.$$

$$b_t = (1+r)^{t-65} (N - \Pi) - \sum_{s=1}^{t-65} (1+r)^s [c_{t-s} - (y_{pre} + y_{ann})] \geq 0 \quad \forall t$$

- y_{pre} is pension income, Π is annuitized wealth and y_{ann} is annuity income.
- Premium for an annuity paying a constant real stream of y_{ann} is:

$$\Pi(y_{ann}, \lambda) = \sum_{t=65}^T \frac{S_t y_{ann}}{(1+r)^{t-65}} / (1-\lambda)$$

Parameterization

Table 1

Parameters of the model.

Parameter values

Preferences

$$EU = \sum_{t=65}^T \beta^{t-65} S_t u(c_t) + \sum_{t=66}^{T+1} p_t v\left(\frac{b_t}{(1+r)^{t-65}}\right)$$

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \sigma = 2$$

$$v(b) = a \left(\sum_{i=1}^{\infty} \beta^{i-1} \right) \frac{\left(y_h + \frac{b}{\sum_{i=1}^{\infty} (1+r)^{-(i-1)}} \right)^{1-\sigma}}{1-\sigma}, \quad y_h = y_{ref}, \text{ Vary } a \text{ to vary strength of bequest motive}$$
$$\beta = \frac{1}{1.03}$$

Budget set

$W \equiv 1$, Normalization (the problem is scalable)

$N = \frac{W}{2}$, One-half of total wealth is already annuitized

$$y_{ref} = \frac{W - N}{\sum_{t=65}^T (1+r)^{-(t-65)} S_t}$$

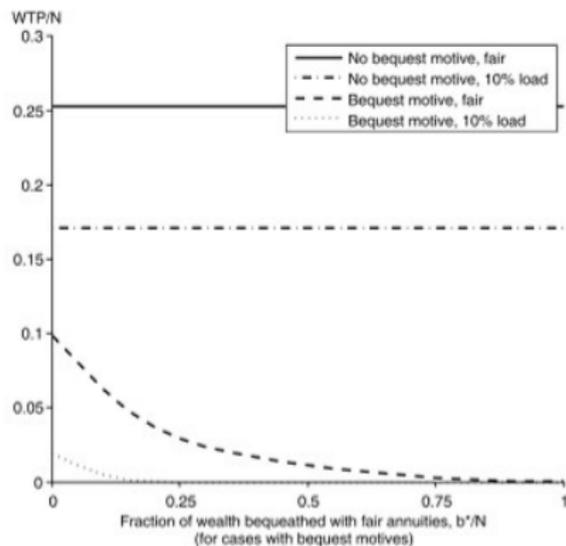
$r = 0.03$

Risk

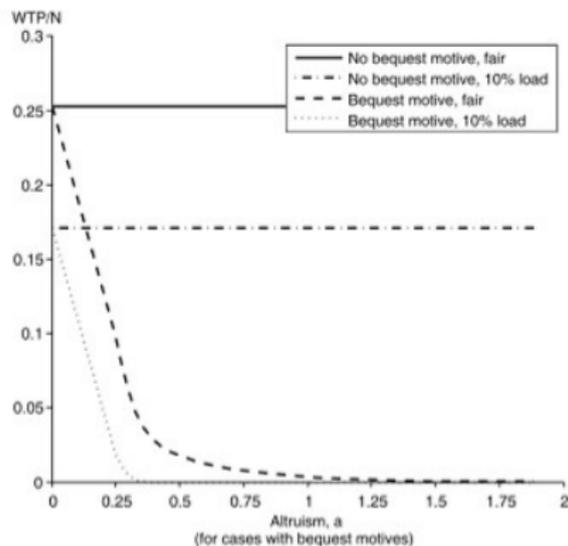
$\{(p_t, S_t)\}_{t=65}^{111}$ from 2003 U.S. Social Security Administration male life table, adjusted so $S_{111} = 0$

Note. Aside from the bequest motive, all parameter values are standard in the annuity literature.

Welfare gains from annuities



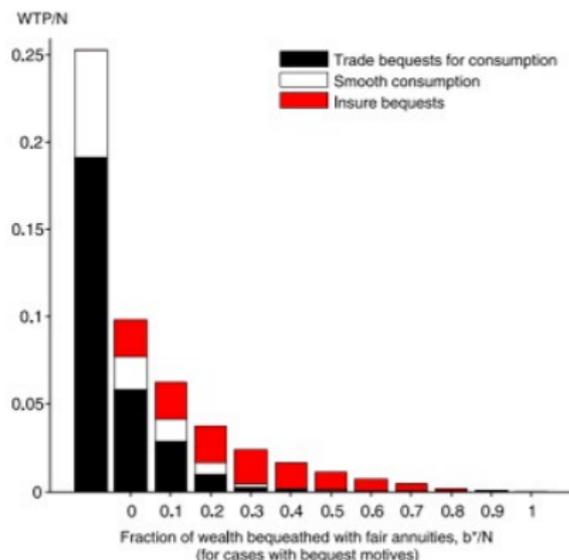
(a)



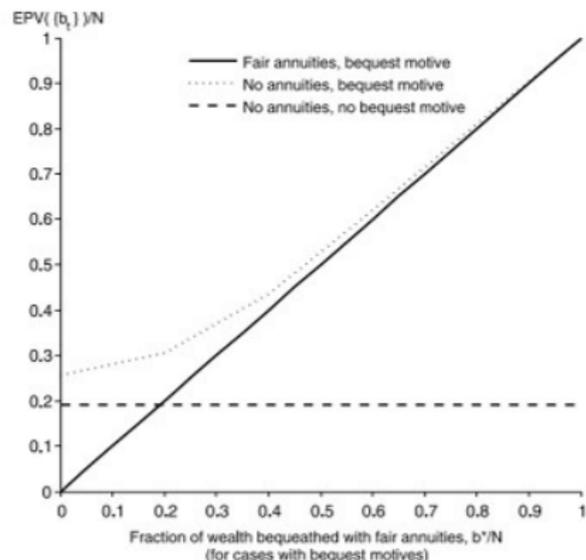
(b)

Fig. 1. Welfare gains from annuities as a function of the strength of the bequest motive. The gain from annuities is measured as the fraction of the individual's non-annuity wealth that he would be willing to pay for access to the annuities. In panel (a), the strength of the bequest motive is measured as the fraction of the individual's non-annuity wealth that he would bequeath had he access to actuarially fair annuities, b^*/N . In panel (b), the strength of the bequest motive is measured as the degree of altruism, a . One-half of wealth is already annuitized, which is roughly the average share among 65-year-olds in the eighth and ninth deciles of the wealth distribution.

Decomposition of welfare gains



(a)



(b)

Fig. 2. Panel (a): Components of the gain from actuarially fair annuities for individuals without bequest motives (first bar) and for individuals with bequest motives of various strengths. Panel (b): Expected discounted bequests as a fraction of initial non-annuity wealth. One-half of wealth is already annuitized.

Simulating with different bequest motives specifications

Table 2

Bequest motives used to calculate the aggregate demand for annuities.

Paper	Bequest motive	Parameters
Ameriks et al. (2009)	$v(b) = \frac{\omega}{1-\sigma} (\phi + \frac{b}{\omega})^{1-\sigma}$	$\omega = 16, \phi = 5.05$
De Nardi (2004)	$v(b) = \phi_1 (1 + \frac{b}{\phi_2})^{1-\sigma}$	$\phi_1 = -9.5, \phi_2 = 11.6$
De Nardi et al. (2010)	$v(b) = \theta \frac{(k+b)^{1-\sigma}}{1-\sigma}$	$\theta = 2, 360, k = 273$
Hurd and Smith (2002) ^a	$v(b) = \theta b$	$\theta = 25.5^{-\sigma}$
Kopczuk and Lupton (2007)	$v(b) = \theta b$	$\theta = 23.8^{-\sigma}$
Lockwood (2010)	$v(b) = (\frac{m}{1-m})^\sigma \frac{(\frac{m}{1-m} c_0 + b)^{1-\sigma}}{1-\sigma}$	$m = 0.96, c_0 = 18$

Notes. Utility from consumption is constant relative risk aversion, $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, in all cases. Unlike the previous sections, preferences are defined over real bequests rather than the present value of bequests because all of the estimated bequest motives come from models in which preferences are defined over real bequests.

^a The Hurd and Smith (2002) bequest motive is one I estimate to match Hurd and Smith's estimates of average anticipated bequests.

Simulating with different bequest motives specifications

Table 4
Simulated and empirical annuity ownership rates among 65–69-year-old single retirees in the Health and Retirement Study.

	Annuity ownership rate			
Bequest motive				
None	69.1%	67.6%	61.0%	54.9%
De Nardi et al. (2010)	60.6%	62.3%	56.4%	44.8%
Hurd and Smith (2002)	24.7%	15.6%	11.7%	4.9%
Kopczuk and Lupton (2007)	21.7%	13.6%	10.6%	4.4%
De Nardi (2004)	19.0%	18.5%	15.4%	4.5%
Lockwood (2010)	16.8%	16.8%	12.6%	3.8%
Ameriks et al. (2009)	0.3%	0.0%	0.0%	0.0%
Average ownership, middle four bequest motives ^a	20.5%	16.1%	12.6%	4.4%
Empirical ownership rate	3.6%			
Model				
Medical spending	No	Yes	Yes	Yes
Minimum annuity size	\$0	\$0	\$20,000	\$20,000
Fixed cost	\$0	\$0	\$0	\$2,000

Note. The simulation uses annuities with a ten percent load, typical of the U.S. private market.

^a Simple average of the ownership rates using the Hurd and Smith (2002), Kopczuk and Lupton (2007), De Nardi (2004), and Lockwood (2010) bequest motives.

Various motives - Paschenko (2013-JPubE)

- Quantitative model to assess several explanations for the lack of annuitization:
 - ① Preannuitized wealth in retirees' portfolios.
 - ② Adverse selection.
 - ③ Bequest motives.
 - ④ Medical expense uncertainty.
 - ⑤ **Government safety net in terms of means-tested transfers.**
 - ⑥ **Illiquidity of housing wealth.**
 - ⑦ **Restrictions on minimum amount of investment in annuities.**
- 1), 3), 6) and 7) play a big role in reducing annuity demand.

The model

- Portfolio choice model of a single retiree.
- Agent chooses how much to save and how to split his net worth between bonds and annuities.
- Uncertain lifespan and out-of-pocket medical expenses.
- Heterogeneity in age, health status, initial wealth, and permanent income.
- Preferences are:

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \quad v(k_t) = \eta \frac{(\phi + k_t)^{1-\sigma}}{1-\sigma}$$

The model

- Medical expenses are modeled following De Nardi et al. (2010):

$$\ln(z_t) = \mu(m, t, l) + \sigma_z \psi_t$$

with

$$\psi_t = \zeta_t + \xi_t, \quad \xi_t \sim N(0, \sigma_\xi^2)$$

$$\zeta_t = \rho_{hc} \zeta_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2)$$

- If affording medical expenses is not affordable there is a transfer of the government τ_t (guarantees c_{min}).

The model - Recursive problem

$$\begin{aligned} V(X_t, m_t, \zeta_t, \epsilon_t) &= \underset{c_t, k_{t+1}, \Delta_{t+1}}{\text{Max}} u(c_t) \\ &+ \beta s_t \Pr(m_{t+1} = 0 \mid m_t, t, I) \int_{\zeta, \xi} V(X_{t+1}, 0, \zeta_{t+1}, \xi_{t+1}) dF(\zeta_{t+1}, \xi_{t+1} \mid \zeta_t) \\ &+ \beta s_t \Pr(m_{t+1} = 1 \mid m_t, t, I) \int_{\zeta, \xi} V(X_{t+1}, 0, \zeta_{t+1}, \xi_{t+1}) dF(\zeta_{t+1}, \xi_{t+1} \mid \zeta_t) \\ &\quad + \beta(1 - s_t)v(k_{t+1}) \\ &\quad \text{s.t.} \\ c_t + z_t + k_{t+1} + q_t \Delta_{t+1} &= k_t(1 + r) + n_t + \tau_t \\ \tau_t &= \min\{0, c_{\min} - k_t(1 + r) - n_t + z_t\} \\ n_{t+1} &= \Delta_{t+1} + n_t \\ k_{t+1}, \Delta_{t+1} &\geq 0 \end{aligned}$$

The model - Insurance sector

- Annuity contracts are non-exclusive and linear.
- Restriction on the minimum amount that can be invested in annuities equal to $\bar{\Delta}$.
- Maximum issue age for annuities \bar{t} .
- Expected payout per unit of insurance sold to an individual of age is:

$$\pi_t(\Omega_t) = q_t(\Omega_t) - \gamma \sum_{i=1}^{T-t} \frac{\hat{S}_{t+i \setminus t}(\Omega_t)}{(1+r)^i}$$

$\gamma \geq 1$ is the administrative load, Ω_t is the set of information available to an insurer about an individual of age t .

- Choose amount of annuity to sell N_t to maximize $N_t \pi_t$.

Calibration

Table 1
Parameters of the model.

Parameter	Value
Risk aversion σ	3.84
Discount factor β	0.97
Strength of bequest motive η	2360
Shift parameter ϕ	\$273,000
Interest rate r	2%
Administrative load γ	10%
Consumption floor c_{\min}	\$2665
Maximum issue age \bar{t}	88 years
Minimum purchase $\bar{\Delta}$	\$2500
Persistence ρ_{hc}	0.849
Variance of medical costs σ_z^2	1.78
Variance of transitory shock σ_{ε}^2	0.524
Variance of persistent shock σ_{ε}^2	0.133

Accounting for non annuitization - all factors

Table 3

Participation in the annuity market: data, model with no impediments to annuitization, and model with all impediments to annuitization.

Income quintile	Data	No impediments to annuitization	All impediments to annuitization
All	5.0	96.6	20.3
1	0.4	82.8	7.0
2	1.0	100.0	14.6
3	5.3	100.0	26.2
4	6.4	100.0	30.7
5	12.2	100.0	23.2

Accounting for non annuitization - Decomposition

Table 4
Annuity market participation rates for the modifications of the full model (top panel) and the simple model (bottom panel).

Income quintile	Full model	No adv. selection	No med. expense	Low c_{\min}	No bequest	Liquid housing	No min. purchase	No preann
All	20.3	19.9	23.6	27.2	36.3	39.0	35.8	42.0
1	7.0	10.5	10.1	8.8	8.1	18.7	12.1	3.9
2	14.6	20.6	20.9	15.8	17.4	32.7	29.3	6.9
3	26.2	33.7	32.9	29.9	34.8	50.3	48.5	29.0
4	30.7	30.1	33.3	43.6	53.5	55.0	51.5	75.7
5	23.2	5.0	20.9	38.1	67.7	38.5	37.8	94.7
Income quintile	Simple model	+ adv. selection	+ med. expense	High c_{\min}	+ bequest	Illiquid housing	+ min. purchase	+ preann
All	96.9	92.2	92.8	84.1	96.6	95.7	95.1	83.5
1	82.8	60.9	65.5	32.9	82.8	78.5	75.8	59.6
2	100.0	100.0	98.7	88.2	100.0	100.0	100.0	77.4
3	100.0	100.0	100.0	99.7	100.0	100.0	100.0	89.8
4	100.0	100.0	100.0	100.0	100.0	100.0	100.0	95.9
5	100.0	100.0	100.0	100.0	100.0	100.0	100.0	94.6

Summary of the results

- Preannuitized wealth, illiquid housing, minimum purchase requirements and bequests are quantitatively important.
- In absence of any impediments to annuitization all but the poorest retirees buy annuities.
- With all impediments to annuitization, the demand for annuities decreases almost five times.
- Adverse selection decreases the demand for annuities among people in the bottom income quintiles but increases the demand in the top.
- With restrictions of minimum amount of annuities, substantially decrease of number of retirees in the market.

Conclusions

- Under a very general setting, a significant wealth annuitization results optimal when there is mortality risk.
- Empirical evidence shows that very few individuals annuitize their wealth.
- Different mechanisms to explain this mismatch between the predictions of a standard life-cycle model and the annuity take-up rate. For example: bequest motives, adverse selections, etc.